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ORIGINAL

MECHANIC-MATHEMATICAL MODEL OF JAVELIN FLIGHT

MODELACIÓN MECÁNICO-MATEMÁTICA DEL VUELO DE LA JABALINA

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ABSTRACT

With a view to improving the athlete's performance during the javelin throw, the objective was to develop, based on the application of Newtonian mechanics to the javelin flight process, a mathematical model and a computerized tool that make it possible to determine the influence on javelin flight trajectory and distance of the characteristic parameters of throwing release phase. As a result, a mathematical model and software were obtained that make it possible to predict the trajectory and flight distance of the implement in function of the properties of the javelin and ambient air, gravitational forces and initial throwing parameters. The model was validated by comparing the flight distance predicted by the model with experimental results obtained in the framework of this research, as well as with data obtained from international competitions. The mean prediction error obtained during the comparison of the javelin flight distance calculated with the model, with the actual throwing results, ranged from 0.65% to 1.58%.

KEY WORDS: javelin, trajectory, mathematical modeling.

RESUMEN

Con vistas al perfeccionamiento del desempeño del atleta durante el lanzamiento de la jabalina, se trazó como objetivo elaborar, sobre la base de la aplicación de la mecánica newtoniana al proceso de vuelo de la jabalina, un modelo matemático y una herramienta computarizada que posibiliten determinar la influencia sobre la trayectoria de vuelo y alcance de la jabalina de los parámetros del lanzamiento en la fase de liberación del implemento. Como resultado se obtienen un modelo matemático y un software que posibilitan predecir el alcance del implemento en función de las propiedades de la jabalina y del aire ambiente, así como de las fuerzas gravitacionales y de los parámetros iniciales del lanzamiento. El modelo fue validado comparando la predicción con resultados experimentales obtenidos en el marco de esta investigación, así como con datos obtenidos de competencias internacionales. El error medio de predicción del alcance de la jabalina, osciló, para diferentes parámetros iniciales, entre 0,65% y 1,58%.

PALABRAS CLAVE: jabalina, trayectoria, modelación matemática.

INTRODUCTION

At present, obtaining better performances from athletes in the athletic modalities related to throws, whether discus, hammer or javelin, is conditioned to the application, during the selection of the initial throwing parameters, of the laws of physics that describe the interaction of the implement with the athlete, with gravitational forces and with the surrounding air.

Analyzes carried out related with athletes of highest performance (Campos, Brizuela, and Ramón, 2004), in which the Cuban javelin thrower Emeterio González was included, during the IAAF World Athletics championship held in 1999, reflected deficiencies in the release parameters during the throwings executed by this athlete, that prevented him from being included in the medal table, despite having given a high initial speed to the implement. These deficiencies can be anticipated and avoided during training sessions by applying a set of scientific results emanating from the application of the laws of classical mechanics to the athlete-implement-gravity-air interaction process that govern the javelin flight process.

From reported results related to javelin throwing, it is evident that it is not possible to fully investigate the influence of the different parameters involved in the throw and flight of the implement, if only experimental studies are carried out. In this type of sporting event, it is practically impossible to execute an experimental design in which certain variables can be fixed in order to study the influence of others, since the athlete is not a machine in which a certain parameter can be fixed with precision. This situation has led researchers to develop theoretical models based on the application of the laws of physics and mathematics, which make it possible, usually with the help of computers, to unravel the essence of the javelin throwing and flight process.

White (2013) suggests that conducting a sensitivity analysis during throwing events, particularly in the case of the javelin, is only possible using a mathematical model. He argues that in practice, it is not possible to request the athlete to perform repeated throws, setting the release angle and applying different release speeds to the implement. He suggests that by means of a model and a computer, tens or thousands of (virtual) throws can be made in a short time, and the input and output parameters can be perfectly controlled. White (2013) also points out that the optimal relationship between the throwing angle and the projection velocity of the implement, with a view to reaching the greatest flight distance, may be particularly different for each athlete and that the determination of this indicator is not possible without the help of mathematical modeling in conjunction with experimentation. This is why the way of conceptual modeling as a research method has been approached by an important group of specialists (Hatton, 2007; Maryniak *et.al*., 2009; Chiu, 2009). The throwing and flight of the javelin is a process governed fundamentally by the laws of mechanics, so it has been the preference of scientists dedicated to this subject the development of mathematical models based on these laws, which make it possible to unravel the interrelationships between the different input and output variables of said process.

Morris and Bartlett (1996) established a reference model for throwing the javelin composed of three hierarchical levels, placing at the first level the one related to the initial parameters of the throw (release parameters), which decisively influence the javelin flight trajectory and distance. Related to this first hierarchical level, several researchers from various countries such as United Kingdom (Hatton, 2007), Poland (Maryniak; Ładyżyńska-Kozdraś & Golińska, 2009) and Taiwan (Chiu, 2009), among others, have developed mathematical models directed to reveal the relative importance of each of these parameters in implement flight trajectory and distance and, in this way, direct the process of improving the throw with a strong scientific support base.

Hatton (2005) elaborates a conceptual model based on the solution of a system of differential equations that incorporate the three elements involved in the movement of the javelin: drag, lift and rotation. The model takes into account the intensity and direction of the wind and was calibrated against data from two international competitions, achieving predictions in the flight distance with an error within 1% for throws between 55-87 m, although making certain assumptions regarding at wind speed, which was not recorded.

Maryniak *et.al*. (2009) develop a model of the man-javelin system together with a mathematical model of the javelin flight, including the effect of transverse elastic vibrations. They perform numerical evaluations of the model for different release speeds and angles, although they do not report a validation of the model based on its comparison with experimental data.

Chiu (2009) elaborates a model and a computerized method to establish optimal conditions of the javelin throw and compares the results of the application of the model with experimental data obtained by Best *et. al.* (1993) and by Mero *et. al.* (1994), obtaining a mean relative error between 4.2% (with the Apollo Olympic New Rules javelin) and 6.8% (with the Held New Rules javelin) when comparing the calculated and measured flight distances of the implement. The applied numerical model considers the prevailing wind conditions, as well as the drag and lift forces acting on the javelin and the position of the pressure center in relation to the implement's center of gravity. The drag and lift forces used by Best *et. al.* (1993) are determined from expressions previously formulated by Soong (1982), although the origin of the values of the drag coefficients is not explained.

Jiang and Zhou (2014) propose a mathematical model that describes the flight of the javelin, obtaining the data referring to the drag forces, lift and the angular or throw moment (pitching moment) from tests in a wind tunnel. In their proposal they expose the optimization calculations carried out by means of a computer, giving as a result that when the initial throw speed is in the range of 25 m / s – 30 m / s, the best release angle is $\theta o = 40^{o}$ (θo - angle of the absolute speed vector of the javelin with the horizontal at the moment of release of the implement), with an angle of attack $\alpha_o = 11^{o}$ (α_o - angle between the longitudinal axis of the javelin and the relative velocity vector between the javelin and the air at the instant of implement release). At the same time the authors define that an angle of attack different from zero turned out to be a necessary and sufficient condition to obtain aerodynamic efficiency in the flight of the javelin.

From the bibliographic analysis carried out, it is evident that experimentation by itself does not constitute a sufficient way of investigation of the javelin throwing process, being necessary to complement the experimental method with the develop and application of a conceptual mathematical model that allow unraveling the essence of the phenomena involved in this process. A mathematical model of this nature, duly validated, which makes it possible to predict the characteristics of the trajectory and flight distance of the javelin for different combinations of initial release parameters of the implement, constitutes a tool of great value during the training of athletes, in particular during the correct selection of throwing parameters during the implement release phase. Although the development of mathematical models that describe the javelin throwing and flight process is reported in the literature, those that are available (Hatton, 2005) only offer as output the prediction of the flight distance of the implement, not making it possible to determine all the variables required in the studies and the level of the input parameters is also limited, so they cannot be applied in smaller categories. In this way, the objectives set out below are outlined for this research.

OBJECTIVES

- To develop, based on the application of the laws of Newtonian Mechanics, a conceptual model that makes it possible to predict the javelin flight trajectory and distance, in function of: the parameters that characterize the release stage of the implement throw; the gravitational forces and the magnitude and direction of prevailing wind;

- To validate the developed model, by comparing the flight distance calculated by the model with the real distance obtained from experimental measurements and with prediction results from other reported models.

THEORETICAL BASIS OF THE MODEL

For the mechanical-mathematical modeling of the javelin flight, we start from the free-body diagram of the implement (Fig. 1) at an instant of time t, corresponding to a position (*x, y*) of its flight path (trajectory of the center of gravity *c.g.*) represented in the figure by a dashed blue line.

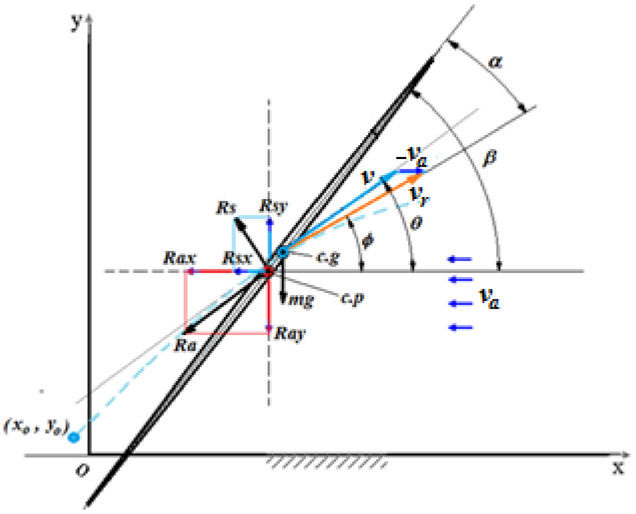


Fig.1 Javelin free body diagram.

The following parameters are represented in the figure (units in the International system, SI):

x-o-y - Cartesian coordinate system fixed to the ground and containing the plane of the trajectory of the javelin, where o-x is the surface of the ground;

c.p. - center of pressures;

a_{cp} - distance from the center of pressures to the center of gravity;

v - absolute speed (tangent to the trajectory) of the center of gravity of the javelin;

v_a - horizontal component of the velocity of the ambient air stream in the x-o-y plane of the javelin trajectory;

v_r - relative velocity between air and javelin in the x-o-y plane;

α - angle of attack of the javelin (angle between the longitudinal axis of the javelin and the relative velocity of the javelin with respect to the air);

β - attitude angle of the javelin (angle formed by the central axis of the javelin with horizontal);

θ - trajectory angle of the javelin (angle formed by the absolute velocity of the javelin with the horizontal);

ϕ - angle with the horizontal of the relative velocity of the javelin with respect to the air);

R_a - aerodynamic drag force offered by the air to the displacement of the javelin, which is applied at the center of pressures (*c.p.*) and acts in the opposite direction to the relative speed v_r;

R_{ax}; R_{ay} - components of the air resistance force on the x and y axes;

R_s - aerodynamic lift force, which is applied at the center of pressures (*c.p.*) and is perpendicular to the direction of relative velocity v_r;

R_{sx}; R_{sy} - components of the lift force in the x and y axes;

m - mass of the javelin;

g - acceleration of gravity ($g=9.8$ m/s^2);

x_o , y_o - coordinates of the javelin release point.

The differential equations of the javelin flight dynamics can be determined based on the application of Newton's 2nd Law to the javelin's free body. In order to simplify the model and therefore the mathematical treatment for its solution, some restrictions that are set out below have been taken into account during the idealization of the phenomenon.

The first restriction is associated with not considering the effect of the moment of forces acting on the javelin in the plane of its trajectory. From previous studies (Hatton, 2005; Maheras, 2013) it is known that during the flight of the javelin the center of pressure does not coincide with the center of gravity, causing a resulting moment of forces called “pitching moment”. This moment causes an angular acceleration that is variable and of an alternative nature, since in addition, the distances between the center of pressures and the center of gravity are variable (González-Martínez, *et. al*., 2019). This variable angular acceleration causes the javelin to rotate in the plane of its trajectory and contributes to flight stability. In the proposed model, the effect of this rotation on javelin trajectory and flight distance is neglected. For this, it is assumed that the effects of the oscillation towards one side of the position that the javelin would have if it did not oscillate, are canceled out with the effects towards the other side of said position. On the other hand, in a previous work by the authors (González-Martínez, *et. al*., 2019) it was determined that the maximum rotational moments that occur in the javelin do not exceed 0.2 N.m.

Another restriction that is assumed during the idealization of the phenomenon is relative to neglecting the effect of the elastic vibrations of the javelin on its trajectory and flight distance. According to Maryniak *et.al*. (2009) this vibratory effect does not have a decisive influence on the trajectory and flight distance of the implement.

A third restriction consisted in not considering the possible effect on the javelin trajectory and flight distance of the precessional movement that the javelin could make in transversal direction to the theoretical plane of flight *x-o-y*.

As a result of these restrictions, during the application of Newton's 2nd Law, only the equations that describe the relationships between the forces and accelerations contained in the plane of the trajectory of the javelin are considered, which are expressed as:

$$Ra_x + Rs_x = m \cdot \frac{dv_x}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

$$m \cdot g + Ra_y - Rs_y = m \cdot \frac{dv_y}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

The components Ra_x and Ra_y are determined based on the aerodynamic drag force R_a which is expressed as (White, F.M., 2011; Golf, J.E., 2013):

$$R_a = \frac{1}{2} \cdot C_a \cdot S_s \cdot \gamma \cdot v_r^{\ 2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

where:

C_a – non-dimensional aerodynamic drag coefficient for the javelin, which varies as a function of the angle of attack ($C_a = f_1(\alpha)$);

γ – air density;

S_s – area of the longitudinal section of the javelin.

The components Rs_x and Rs_y are determined starting from the aerodynamic lift force *Rs* of the javelin, which is expressed as (White, F.M., 2011; Golf, J.E., 2013):

$$R_s = \frac{1}{2} \cdot C_s \cdot S_s \cdot \gamma \cdot v_r^{\ 2} \ldots\ldots\ldots\ldots\ldots\ldots 4$$

being:

C_s – non-dimensional aerodynamic lift coefficient for the javelin, which varies as a function of the angle of attack ($C_s = f_2(\alpha)$).

The functions $C_a = f_1(\alpha)$and $C_s = f_2(\alpha)$, for the javelins of both sexes, are obtained by the authors through the application of Computational Fluid Dynamics tools (González-Martinez, *et. al.*, 2019), resulting in third-order polyomials with coefficiens of determination $R^2 \geq 0{,}99$, given by the following functions:

$$C_a = c_3 \cdot \alpha^3 + c_2 \cdot \alpha^2 + c_1 \cdot a + c_0 \text{ ....5}$$

$$C_s = c_3 \cdot (\alpha+5)^3 + c_2 \cdot (\alpha+5)^2 + c_1 \cdot (\alpha+5) + c_0 \text{ ....6}$$

The values of the coefficients c_i are provided in Table 1, where *h* y *f* corresponds to the male and female sexes respectively.

Table 1. Coefficients of functions *Ca = f1(□) y Cs = f2(□)* for the javelins of both sexes.

	c_3	c_2	c_1	c_0
C_{ah}	$-2 \cdot 10^{-6}$	$2.42 \cdot 10^{-4}$	0	0.077
C_{af}	$-1.4 \cdot 10^{-6}$	$1.80 \cdot 10^{-4}$	0	0.075
C_{sh}	$-2 \cdot 10^{-6}$	$2.35 \cdot 10^{-4}$	0.001	0
C_{sf}	$-1 \cdot 10^{-6}$	$1.20 \cdot 10^{-4}$	0.0025	0

The angle ϕ that forms with the horizontal axis the relative velocity of the javelin with respect to air velocity is given by:

$$\phi = \tan^{-1}\left(\frac{v \cdot sen\theta}{v \cdot \cos\theta - v_a}\right) \text{ ....7}$$

and the angle θ that forms the absolute velocity of the javelin with respect to the horizontal is given by:

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) \text{ ....8}$$

while:

$$\alpha = \beta - \phi \text{ ....9}$$

The coordinates of the trajectory of the center of mass of the javelin are determined as:

$$x = x_o + \int v_x \cdot dt \text{ ....10}$$

$$y = y_o + \int v_y \cdot dt \quad \text{.......} 11$$

In this way, a system of equations is formed whose solution allows the determination of all the main parameters inherent to the javelin fligh (Ra, Rax, Ray, Rs, Rsx, Rsy, v, v_x, v_y, v_r, v_{rx}, v_{ry}, α, θ, ϕ, x, y).

For the solution of the system of equations, a computer program supported in Mathcad and based in the application of Runge-Kutta method was developed. In this way, the expressions are evaluated for small time intervals Δt, whose duration is selected in the program itself. Between these time intervals, a movement with constant acceleration is assumed, which facilitates the treatment of the expressions.

MATERIALS AND METHODS USED FOR THE VALIDATION OF THE THEORETICAL MODEL

The validation of the theoretical model for the prediction of the trajectory and flight distance of the implement was carried out in two ways:

– Evaluating the developed model with the input data from throwing's experiments carried out as part of this research and then comparing the flight distance of the experimental throws with those calculated by the developed mathematical model and by others models reported in the literature;

– Evaluating the model with input data corresponding to throws performed in international competitions (Best et. al, 1993) and comparing the output results of the model with the flight distance obtained in said throws, as well as comparing with the results obtained with other models reported in the literature.

The experimental research was carried out in the sports area of the Faculty of Physical Culture, attached to the Agrarian University of Havana. For the execution of the throws planned in the experimental design, two male students of 4th and 5th course of the multi-event specialty, specialized in javelin throwing were selected. In total 15 throws were executed, in which were determined the following parameters:

As independent variables (Fig. 2) the followings were considered:

– Javelin release velocity: v_o ;

– Initial angle of the trajectory (release angle): θ_o ;

– Initial attack angle: α_0 ;

– Release height (distance from the ground to the center of gravity of the implement at the moment it loses contact with the athlete's hand): y_o ;

– Horizontal distance between the release point and the foul line: x_o, m.

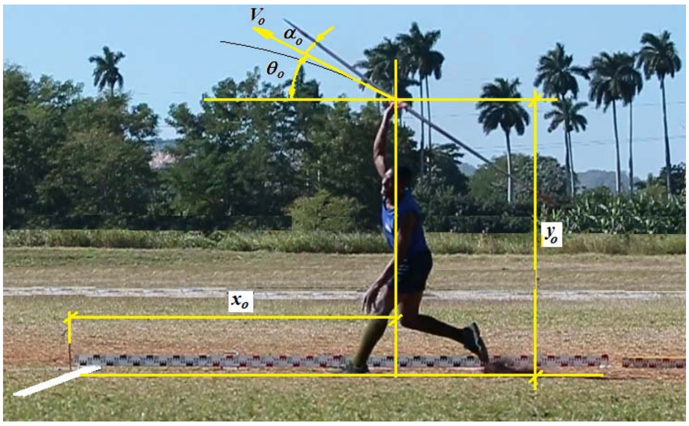


Fig. 2. Independent variables to be determined by image processing

The main dependent variable was the implement's flight distance (x_f) measured from the foul line to the tip of the javelin at the conclusion of its flight path.

As control variables the followings were determined:

– Magnitude and direction of the component of air velocity in the plane of the javelin flight path, measured at a mean height of said path: v_a;

– Mass of the javelin used in the experiments: *m*;

– Position of the center of gravity of the javelin (distance between the *c. g.* and the tip of the javelin): *Lcg*;

– Acceleration of gravity: *g*.

To determine the independent variables, which ultimately constitute the initial throwing parameters, the optical method of motion capture (Pueo y Jiménez, 2017), was applied. For this, digital films were taken from the 15 throws that made up the sample under investigation, for subsequent analysis with computerized image processing means.

For filming, a Canon EOS 70 D video camera was used with a 70.2 mm objective, placed at a filming distance L = 10 m, as shown in Fig. 3. The filming was carried out with a resolution of 1280 X 720 at 60 frames per second, with a hiding speed of 1/4000 seconds. To determine the scale during image processing, images of topography graduated rulers were taken in horizontal and vertical positions prior to the filming. The films were processed in the Laboratory of Educational Technologies of the Agrarian University of Havana (LATED) using the Adobe After-Effect CC 2017 software, obtaining the images of each frame for subsequent processing in an image editor, using in this case the software Windows Paintbrush.

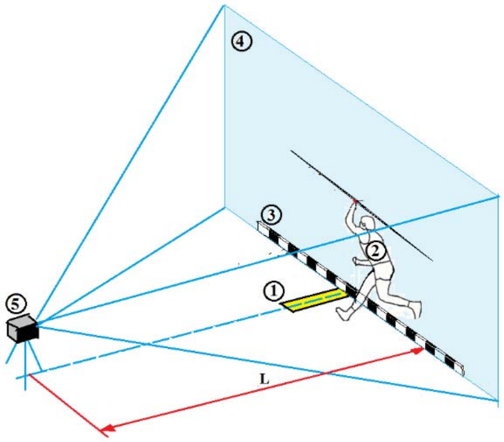
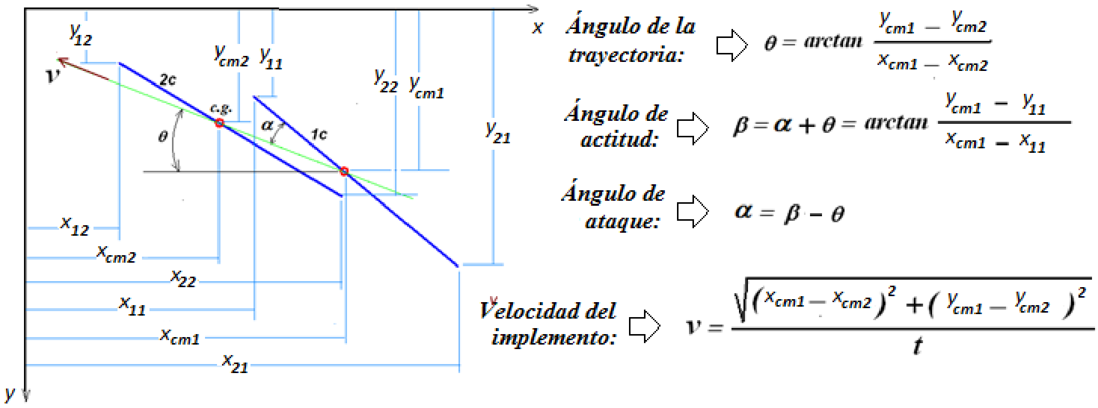


Fig. 3. Scheme of the filming of the throwing events.1) foul line; 2) athlete; 3) graduated scale; 4) filming field; 5) digital video camera; L: distance from the camera to the throwing plane.

On the images corresponding to the frames under study, imported from paintbrush software, the significant coordinates are located (Fig. 4). The values of these coordinates are taken to an Excel spreadsheet where the equations that make it possible to determine the initial throwing parameters are programmed, using the expressions shown in Fig. 4.

The distance reached by the implement (dependent variable) was measured directly in the throwing field, using a measuring tape with a smaller division of 1 cm. The distance was measured from the foul line to the point of contact of the javelin tip with the ground at the end of the flight path.

The component of airspeed on the flight plane was measured with a PROVA AVM-01 digital anemometer with accuracy up to 0.1 m / s. For the measurement, the air speed sensor was placed with its axis of rotation parallel to the direction of the horizontal axis contained in the plane of the implement's flight and at a height from the ground between 6 and 10 m. To prevent any accident, the wind velocity readings were taken immediately before and after each throw, and these values were subsequently averaged.



Ángulo de la trayectoria: ⇨ $\theta = \arctan \dfrac{y_{cm1} - y_{cm2}}{x_{cm1} - x_{cm2}}$

Ángulo de actitud: ⇨ $\beta = \alpha + \theta = \arctan \dfrac{y_{cm1} - y_{11}}{x_{cm1} - x_{11}}$

Ángulo de ataque: ⇨ $\alpha = \beta - \theta$

Velocidad del implemento: ⇨ $v = \dfrac{\sqrt{(x_{cm1} - x_{cm2})^2 + (y_{cm1} - y_{cm2})^2}}{t}$

Fig. 4. Significant coordinates taken during image processing.

In order to achieve a level of variability in the experiments, the athletes were guided to try to execute the throws with different initial implement release angles, as well as with different angles of attack. Likewise, during the throwing execution the air direction was taken into account, both for and against the throwing direction.

RESULTS AND DISCUSSION

A sample of processed frames, corresponding to one of the 15 throws performed with a view to validating the prediction model, is shown in Fig.5. The images correspond to moments before and after the implement release, that is, the short period of time in which it is possible to calculate the initial throwing parameters.

Fig. 5. Succession of frames around the javelin release.

It is interesting to note that given the filming speed, the time elapsed between one frame and another is 1/60 = 16.7 thousandths of a second, enough to “stop” the javelin with sufficient clarity and to be able to specify the characteristic points according to the exposed methodology (Fig. 4). The result of the processing of the data from the selected frames, made it possible, through the application of the expressions of Fig. 4 and their programming in Excel spreadsheets, to build the graphics of the javelin position in *x-o-y* plane (Fig. 6) and determine the initial parameters of the filmed throws.

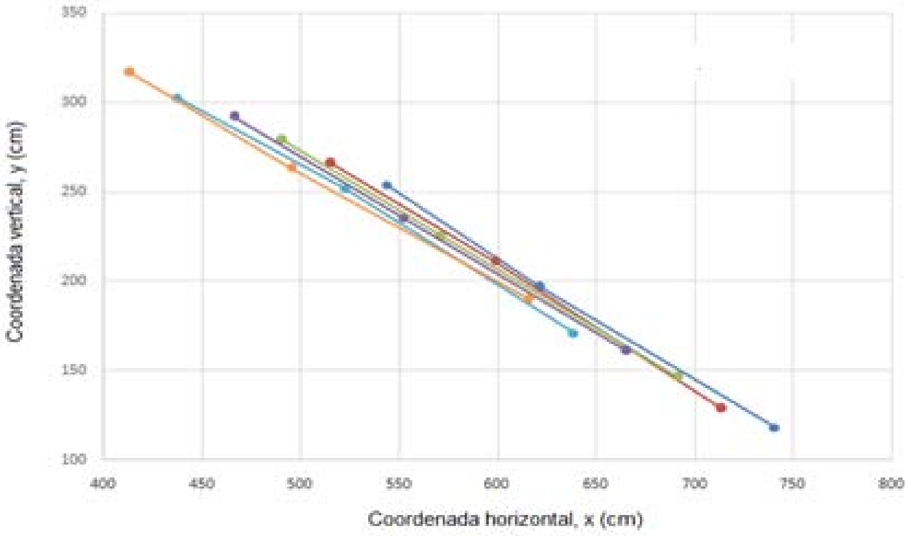


Fig. 6. Graphic result of a sequence of javelin positions in the x-o-y plane near at release instant, corresponding to throw No. 1

A resume of release parameters obtained as a result of frames processing, is shown in Table 2.

Stadigraph	**Release throwing velocity v_o (m/seg)**	**Release throwing angle θ_o (º)**	**Positive attack angle at release □□ (º)**	**Negative attack angle at release □□ (º)**	**High of javelin release point y_o (m)**	**Distance from release point to foul line x_o (m)**
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**
Mean	18,21	32,22	5,40	-4,38	2,10	2,34
Std. Deviation.	1,13	3,53	2,75	4,95	0,10	0,78
Máx. Value	20,17	38,98	8,15	-11,25	2,35	3,76
Min. Value	15,32	23,69	1,79	-0,57	1,90	0,53

Table 2. Summary of the statistics of the initial parameters of filmed throws.

Ambient air velocity, taken in the horizontal direction of the implement's path plane, ranged from -3 m/s (against implement movement) to +1.5 m/s (in favor of implement movement). Finally, the measurement of the flight distance, measured from the foul line, averaged 31.03 m, ranging from 23.22 to 35.65 m.

For the evaluation of the prediction model, the system of equations arising from the analysis of the dynamics and kinematics of the interaction between javelin, air and gravity during flight, was solved through programming in Mathcad, resulting in the software “Javelin” that has been registered in the Copyright Center of the Republic of Cuba (CENDA, 2018 y 2018a). The deterministic nature of the model ensures the absence of uncertainty in the result, that is, for a given set of input variables, the same output result or response is always obtained. As a fundamental output the software provides a graph of the javelin trajectory, as well as tabulated data from which output parameters can be determined, such as: the implement flight distance, the flight time, the maximum height reached by the trajectory, the values of the drag and lift forces as a function of the position of the javelin throughout the trajectory, the angles that characterize the position of the javelin during flight and other data of interest.

In Fig.7 a graphical output of the software "Javelin" is shown. Input data were taken from Campos *et al*. (2004), corresponding to the best perform of the Cuban thrower Emeterio González (84.32 m) during the IAAF World Athletics Championships, Valencia, 1999.

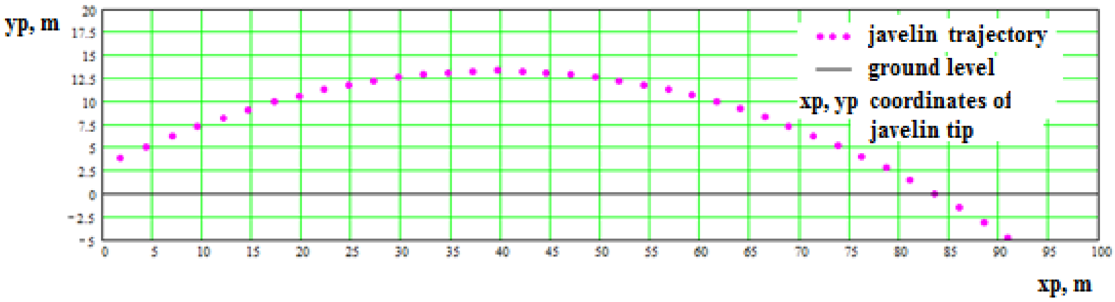


Fig. 7. Output of software “Javelin” representing the javelin trajectory corresponding to best throw performed by Emeterio González in the World Athletics Championships, 1999.

The first validation way was carried out by evaluating the model with the input data obtained from filmed throws and then comparing the flight distance (output or dependent variable) experimentally measured in the 15 performed throws, with flight distances calculated by the model.

Figure 8 shows the results of a regression analysis carried out between the flight distances measured experimentally and the prediction obtained by evaluating the model with the initial data from each throw. From the trend lines obtained, it can be seen that there is a strong relationship between the model predictions and the experimental data, characterized by a high coefficient of determination R^2. A perfect result (almost impossible to achieve, not only from the point of view of the model, but a product of the accuracy of the experimental measurements) would be a slope equal to unity and a correlation coefficient R = 1. It can be seen that although this level is not achieved, the result (y=1,003 x; R^2=0,989) is very satisfactory. The error of the model prediction in relation to the experimental measurement, only in one of the throws was greater than 2%, while the mean modular value of the errors averaged 0.65%.

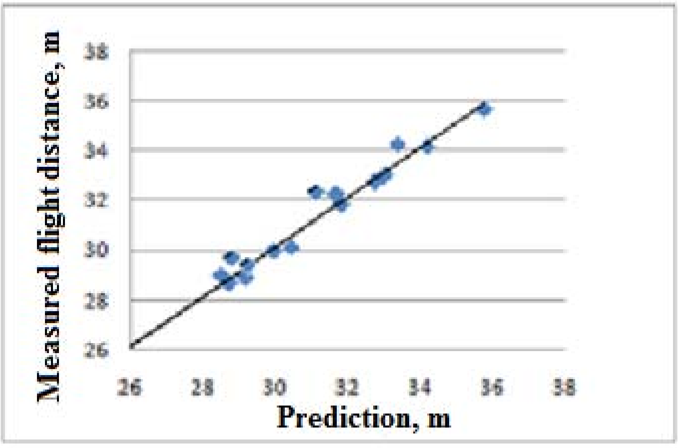


Fig. 8. Correlation between throw distances measured experimentally and calculated using the model and the software “Javelin” (Regression equation: y = 1.003 x; Coefficient of determination: R^2 = 0.989)

A second way of validation consisted of evaluating the model with data from international competitions, as well as with results of the evaluation of other models reported in the literature. For this, a group of initial throws data taken by Best *et. al*. (1993) in the World Student Games held in Sheffield, England, in 1991, corresponding to a selection of the best throws executed in the female modality were evaluated. These data were also evaluated by Chiu (2009) using computerized simulation, so it served as a comparative basis as a validation way of the “Javelin” model presented in this work.

Table 3 shows the results compiled by Best *et. al.* (1993), as well as the results of the evaluation of the “Javelin” model and the Chiu model (2009). Both models present a satisfactory approximation to the real results, however, the errors obtained with the “Javelin” model were lower than those obtained by Chiu (2009).

Throw	**Release throwing velocity v_o (m/seg)**	**Release throwing angle θ_o (º)**	**Attack angle at release □□ (º)**	**High of javelin release point y_o (m)**	**Measured flight distance x_f (m)**	**Flight distance calculated with “Javelin” model Xfc (m)**	**Flight distance calculated with Chiu model Xfch (m)**	**Error with “Javelin” model** (%)	**Error with Chiu model** (%)
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**
Fem1	24,2	34,5	1,5	1,66	57,22	56,66	57,68	-0,979	0,8
Fem2	24,6	39	1	1,68	59,34	60,55	61,33	2,039	3,3
Fem3	24,8	38	0,5	1,62	62,32	61,02	60,70	-1,218	-3,5
Fem4	24,2	33,5	13,5	1,72	58,28	58,99	56,24	2,084	-2,5
							Modular mean error (%)	**1,58**	**2,56**

Table 3. Results of the evaluation of the theoretical models vs. experimental results obtained by Best et al (1993) in world competition of the female modality

Another way of validation consisted of evaluating the available online Hatton model (Hatton, 2005), with input data taken of throwing’s experiments carried out as part of this research, and then comparing the results with those obtained during the evaluation of "Javelin" model with these same input data and taking into account that the Hatton software does not understand v_o values below 18 m/s, or θo values below 30º. In Fig. 9 the comparison of the experimentally measured results with those from the evaluation of both models is shown.

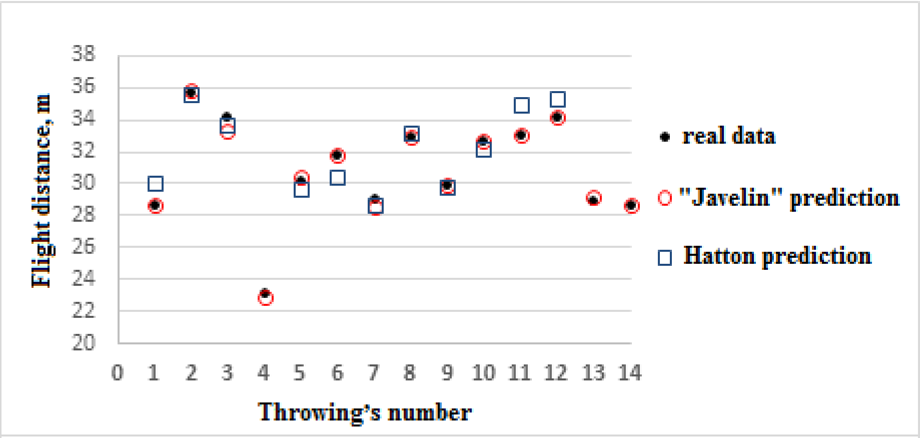


Fig. 9. Comparison of actual implement flight distance with results from the evaluation of "Javelin" and Hatton models.

The figure shows that the results of the "Javelin" model present a better level of approximation to the experimental results in relation to the Hatton model. A statistical analysis of the mean modular prediction error yielded an porcentual error of 0.65% in "Javelin" model versus 2.1% in Hatton model.

CONCLUSIONS

A mechanical-mathematical model of the javelin flight, in its interaction with the surrounding air and with the gravitational force, is obtained. The evaluation of the model with aids of a software developed for this purpose, makes it possible to obtain the main parameters that characterize the flight trajectory of the javelin, in function of input parameters such as the speed and angles of release and attack of the implement and the coordinates of the release point, as well as the inertial and aerodynamic properties of the javelin.

The validation process of the model by comparing the results of the calculated javelin flight distance and that obtained by means of experimental measurements carried out within this research and through the comparison with results of international competitions, yielded a mean prediction error between 0,65% and 1,58%, which was lower than that obtained with other reported models.

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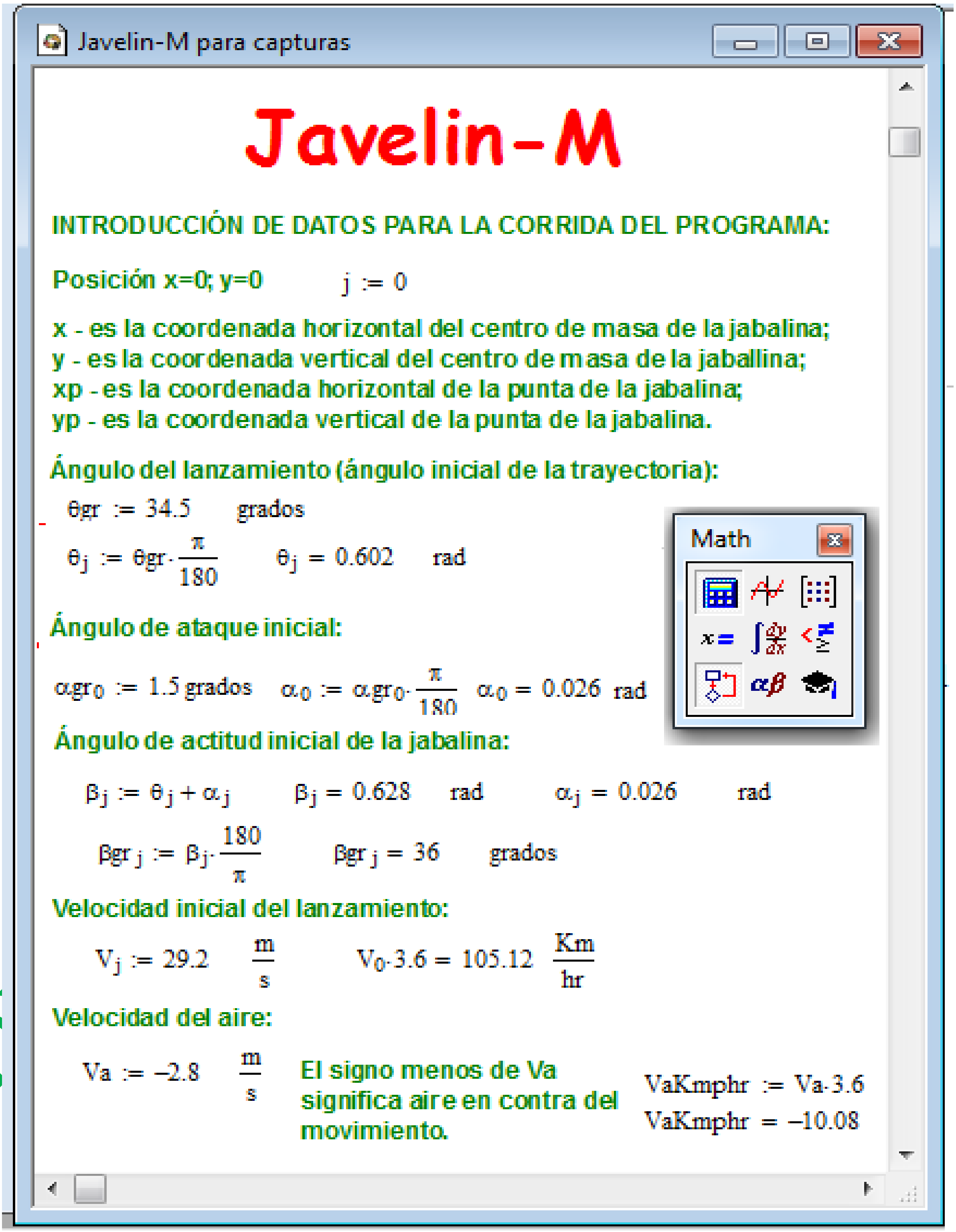
ANEXO 1

1.1 Muestra de capturas de pantalla del programa "Javelin M"



Javelin-M

INTRODUCCIÓN DE DATOS PARA LA CORRIDA DEL PROGRAMA:

Posición x=0; y=0 $j := 0$

x - es la coordenada horizontal del centro de masa de la jabalina;
y - es la coordenada vertical del centro de masa de la jaballina;
xp - es la coordenada horizontal de la punta de la jabalina;
yp - es la coordenada vertical de la punta de la jabalina.

Ángulo del lanzamiento (ángulo inicial de la trayectoria):

$\theta gr := 34.5$ grados

$\theta_j := \theta gr \cdot \frac{\pi}{180}$ $\theta_j = 0.602$ rad

Ángulo de ataque inicial:

$\alpha gr_0 := 1.5$ grados $\alpha_0 := \alpha gr_0 \cdot \frac{\pi}{180}$ $\alpha_0 = 0.026$ rad

Ángulo de actitud inicial de la jabalina:

$\beta_j := \theta_j + \alpha_j$ $\beta_j = 0.628$ rad $\alpha_j = 0.026$ rad

$\beta gr_j := \beta_j \cdot \frac{180}{\pi}$ $\beta gr_j = 36$ grados

Velocidad inicial del lanzamiento:

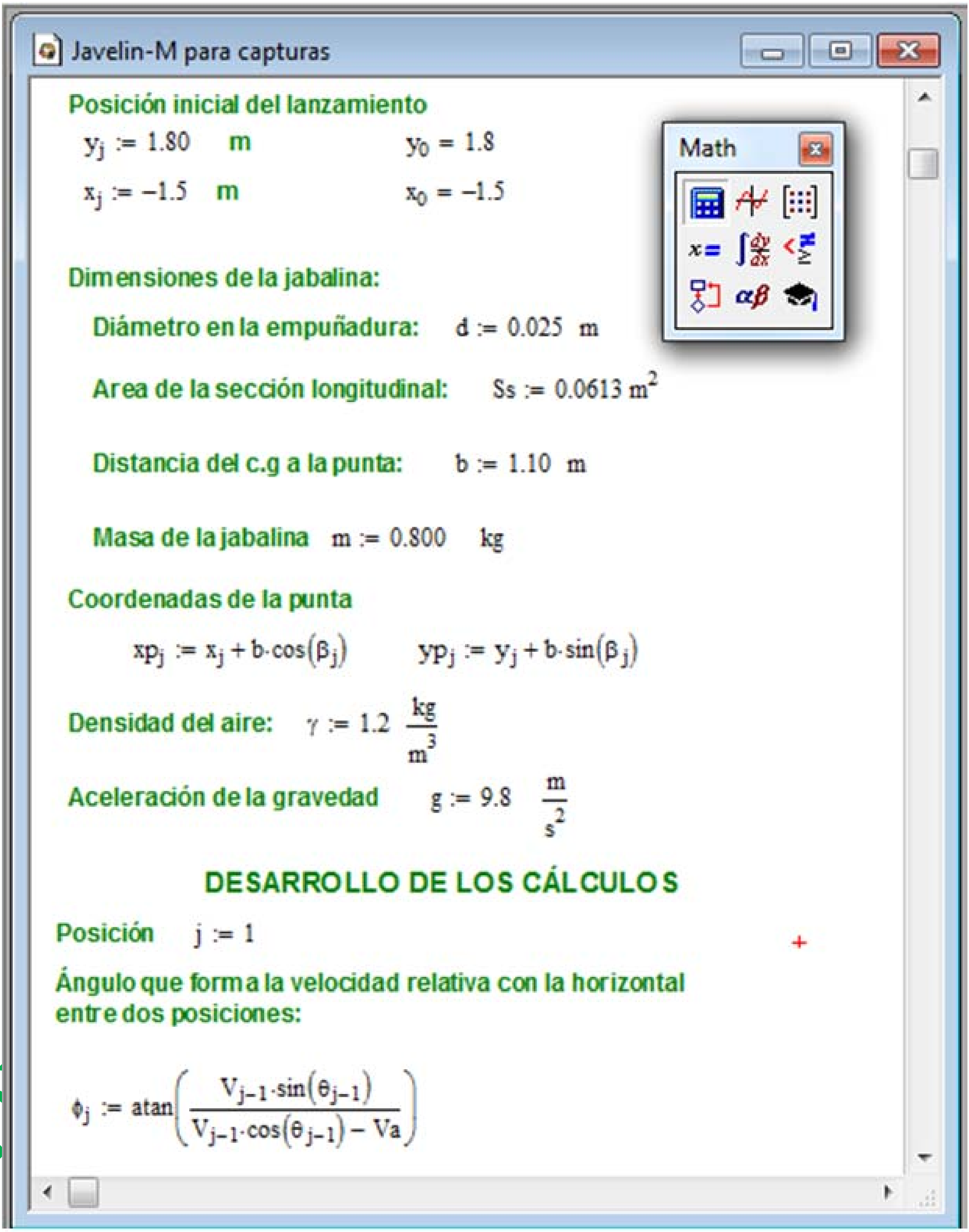
$V_j := 29.2 \ \frac{m}{s}$ $V_0 \cdot 3.6 = 105.12 \ \frac{Km}{hr}$

Velocidad del aire:

$Va := -2.8 \ \frac{m}{s}$

El signo menos de Va significa aire en contra del movimiento.

$VaKmphr := Va \cdot 3.6$
$VaKmphr = -10.08$



Posición inicial del lanzamiento

$y_j := 1.80$ m $\quad y_0 = 1.8$

$x_j := -1.5$ m $\quad x_0 = -1.5$

Dimensiones de la jabalina:

Diámetro en la empuñadura: $d := 0.025$ m

Area de la sección longitudinal: $Ss := 0.0613\ m^2$

Distancia del c.g a la punta: $b := 1.10$ m

Masa de la jabalina $m := 0.800$ kg

Coordenadas de la punta

$xp_j := x_j + b\cdot\cos(\beta_j) \qquad yp_j := y_j + b\cdot\sin(\beta_j)$

Densidad del aire: $\gamma := 1.2\ \frac{kg}{m^3}$

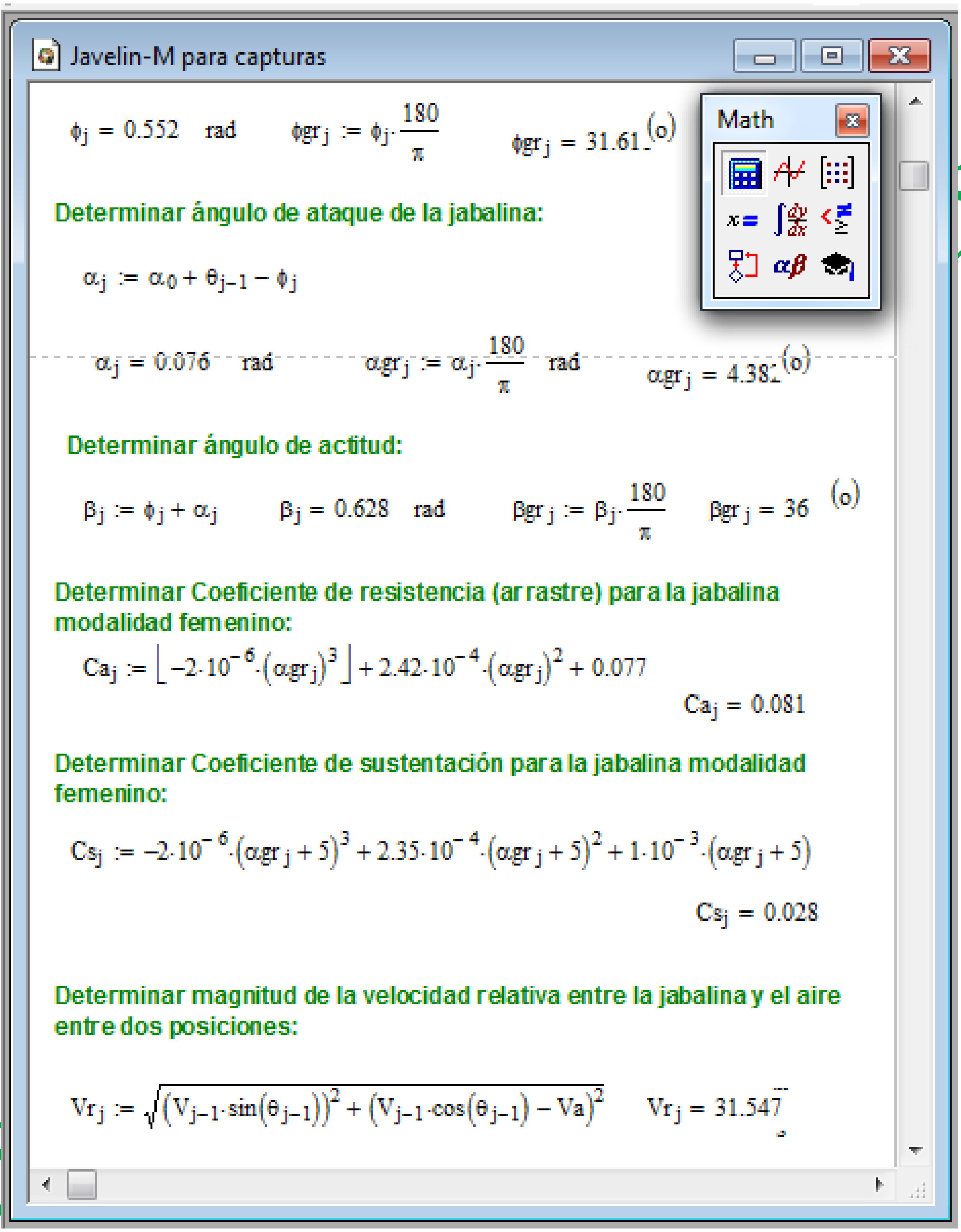
Aceleración de la gravedad $g := 9.8\ \frac{m}{s^2}$

DESARROLLO DE LOS CÁLCULOS

Posición $j := 1$

Ángulo que forma la velocidad relativa con la horizontal entre dos posiciones:

$$\phi_j := \operatorname{atan}\left(\frac{V_{j-1}\cdot\sin(\theta_{j-1})}{V_{j-1}\cdot\cos(\theta_{j-1}) - Va}\right)$$



$\phi_j = 0.552$ rad $\qquad \phi gr_j := \phi_j \cdot \frac{180}{\pi}$ $\qquad \phi gr_j = 31.61$ (o)

Determinar ángulo de ataque de la jabalina:

$$\alpha_j := \alpha_0 + \theta_{j-1} - \phi_j$$

$\alpha_j = 0.076$ rad $\qquad \alpha gr_j := \alpha_j \cdot \frac{180}{\pi}$ rad $\qquad \alpha gr_j = 4.38$ (o)

Determinar ángulo de actitud:

$\beta_j := \phi_j + \alpha_j$ $\qquad \beta_j = 0.628$ rad $\qquad \beta gr_j := \beta_j \cdot \frac{180}{\pi}$ $\qquad \beta gr_j = 36$ (o)

Determinar Coeficiente de resistencia (arrastre) para la jabalina modalidad femenino:

$$Ca_j := \left\lfloor -2\cdot 10^{-6}\cdot (\alpha gr_j)^3 \right\rfloor + 2.42\cdot 10^{-4}\cdot (\alpha gr_j)^2 + 0.077$$

$$Ca_j = 0.081$$

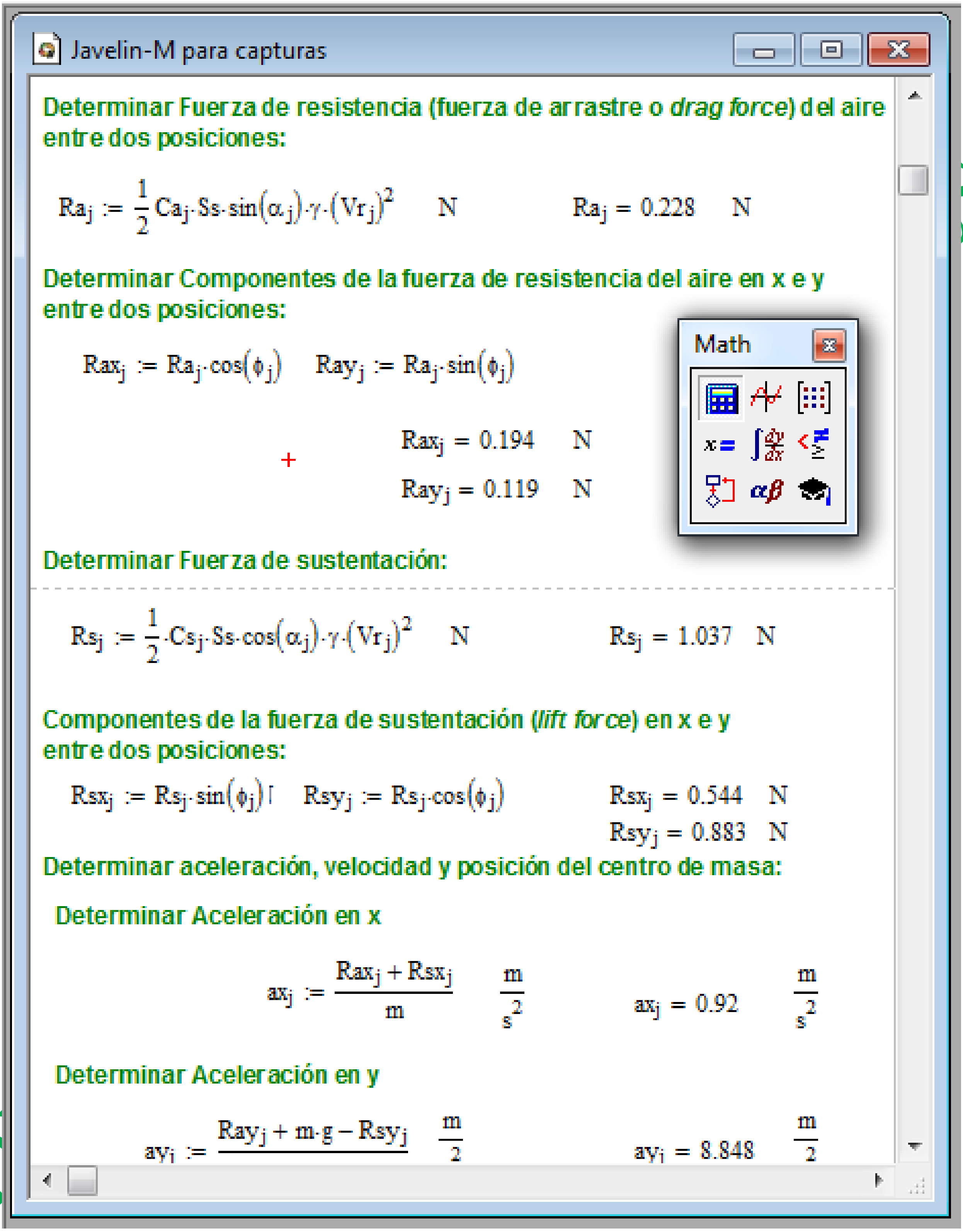
Determinar Coeficiente de sustentación para la jabalina modalidad femenino:

$$Cs_j := -2\cdot 10^{-6}\cdot (\alpha gr_j + 5)^3 + 2.35\cdot 10^{-4}\cdot (\alpha gr_j + 5)^2 + 1\cdot 10^{-3}\cdot (\alpha gr_j + 5)$$

$$Cs_j = 0.028$$

Determinar magnitud de la velocidad relativa entre la jabalina y el aire entre dos posiciones:

$$Vr_j := \sqrt{\left(V_{j-1}\cdot \sin(\theta_{j-1})\right)^2 + \left(V_{j-1}\cdot \cos(\theta_{j-1}) - Va\right)^2} \qquad Vr_j = 31.547$$



Determinar Fuerza de resistencia (fuerza de arrastre o *drag force*) del aire entre dos posiciones:

$Ra_j := \frac{1}{2} Ca_j \cdot Ss \cdot \sin(\alpha_j) \cdot \gamma \cdot (Vr_j)^2$ N $Ra_j = 0.228$ N

Determinar Componentes de la fuerza de resistencia del aire en x e y entre dos posiciones:

$Rax_j := Ra_j \cdot \cos(\phi_j)$ $Ray_j := Ra_j \cdot \sin(\phi_j)$

$Rax_j = 0.194$ N

$Ray_j = 0.119$ N

Determinar Fuerza de sustentación:

$Rs_j := \frac{1}{2} Cs_j \cdot Ss \cdot \cos(\alpha_j) \cdot \gamma \cdot (Vr_j)^2$ N $Rs_j = 1.037$ N

Componentes de la fuerza de sustentación (*lift force*) en x e y entre dos posiciones:

$Rsx_j := Rs_j \cdot \sin(\phi_j)$ $Rsy_j := Rs_j \cdot \cos(\phi_j)$ $Rsx_j = 0.544$ N

$Rsy_j = 0.883$ N

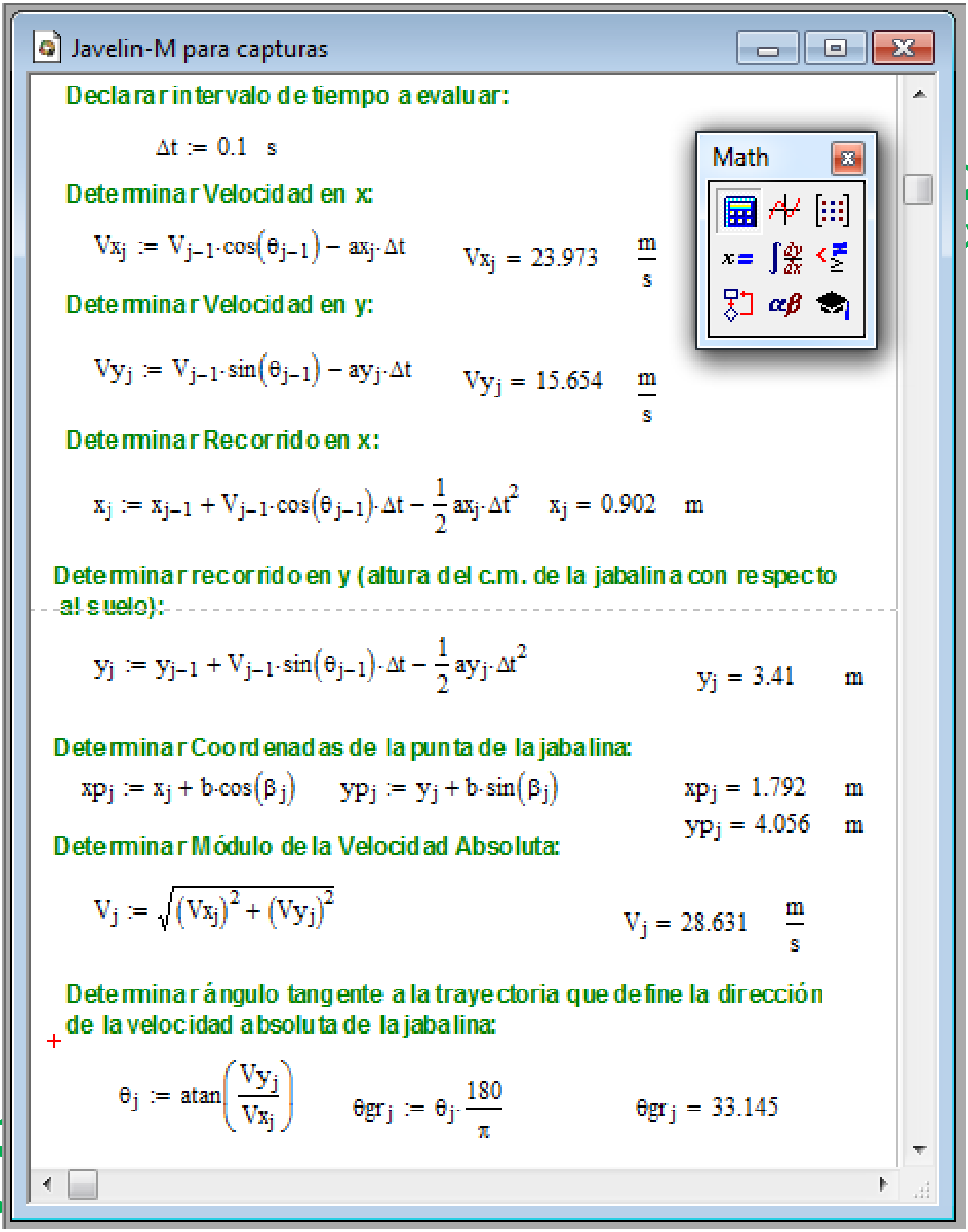
Determinar aceleración, velocidad y posición del centro de masa:

Determinar Aceleración en x

$ax_j := \frac{Rax_j + Rsx_j}{m}$ $\frac{m}{s^2}$ $ax_j = 0.92$ $\frac{m}{s^2}$

Determinar Aceleración en y

$ay_j := \frac{Ray_j + m \cdot g - Rsy_j}{}$ $\frac{m}{2}$ $ay_j = 8.848$ $\frac{m}{2}$



Declarar intervalo de tiempo a evaluar:

$\Delta t := 0.1\ \text{s}$

Determinar Velocidad en x:

$Vx_j := V_{j-1}\cdot\cos(\theta_{j-1}) - ax_j\cdot\Delta t$ $\quad Vx_j = 23.973\ \frac{m}{s}$

Determinar Velocidad en y:

$Vy_j := V_{j-1}\cdot\sin(\theta_{j-1}) - ay_j\cdot\Delta t$ $\quad Vy_j = 15.654\ \frac{m}{s}$

Determinar Recorrido en x:

$x_j := x_{j-1} + V_{j-1}\cdot\cos(\theta_{j-1})\cdot\Delta t - \frac{1}{2}ax_j\cdot\Delta t^2$ $\quad x_j = 0.902\ m$

Determinar recorrido en y (altura del c.m. de la jabalina con respecto al suelo):

$y_j := y_{j-1} + V_{j-1}\cdot\sin(\theta_{j-1})\cdot\Delta t - \frac{1}{2}ay_j\cdot\Delta t^2$ $\quad y_j = 3.41\ m$

Determinar Coordenadas de la punta de la jabalina:

$xp_j := x_j + b\cdot\cos(\beta_j)$ $\quad yp_j := y_j + b\cdot\sin(\beta_j)$ $\quad xp_j = 1.792\ m$

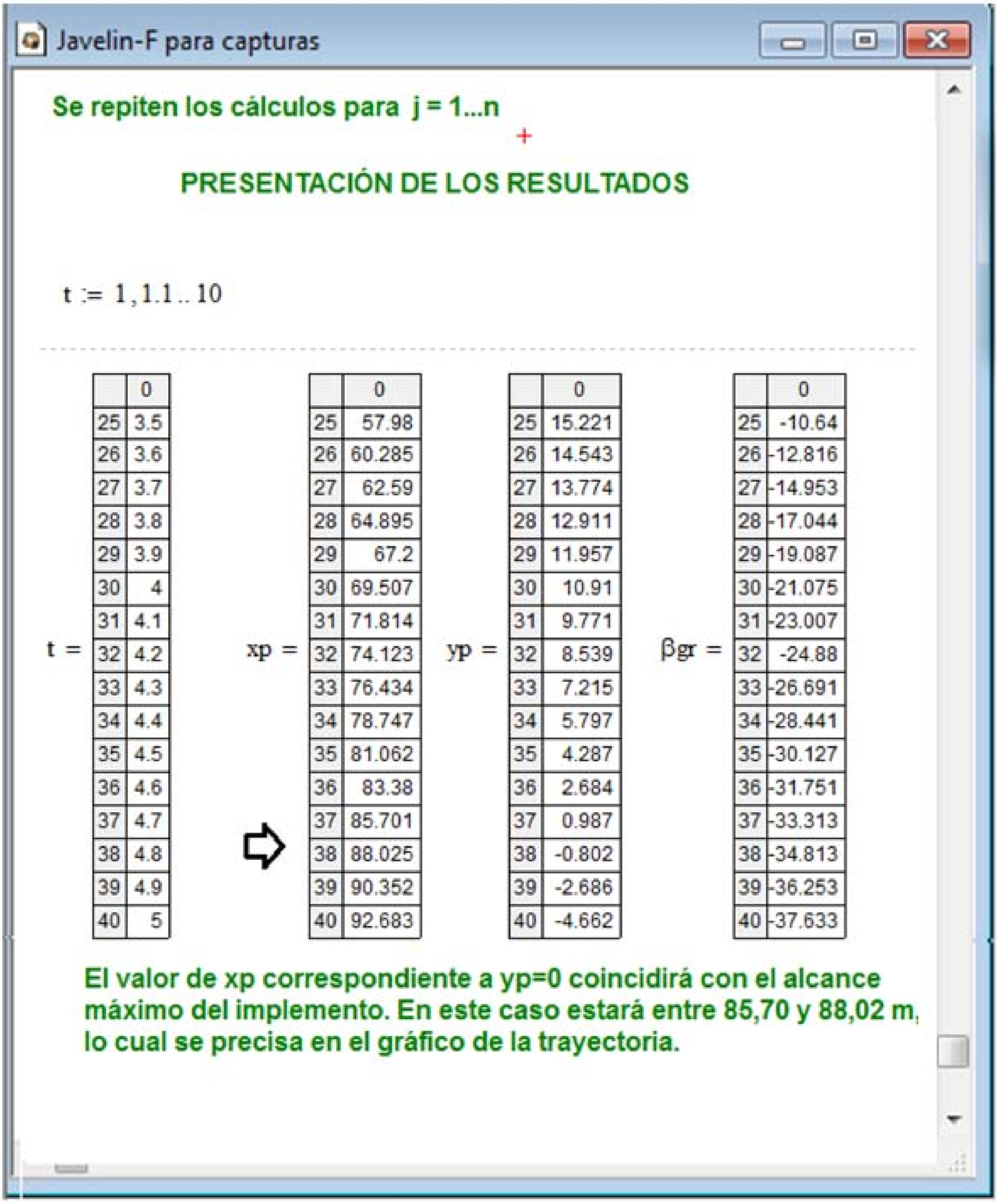
$yp_j = 4.056\ m$

Determinar Módulo de la Velocidad Absoluta:

$V_j := \sqrt{(Vx_j)^2 + (Vy_j)^2}$ $\quad V_j = 28.631\ \frac{m}{s}$

Determinar ángulo tangente a la trayectoria que define la dirección de la velocidad absoluta de la jabalina:

$\theta_j := \operatorname{atan}\left(\frac{Vy_j}{Vx_j}\right)$ $\quad \theta gr_j := \theta_j\cdot\frac{180}{\pi}$ $\quad \theta gr_j = 33.145$



Se repiten los cálculos para j = 1...n

PRESENTACIÓN DE LOS RESULTADOS

$t := 1, 1.1 .. 10$

t =

	0
25	3.5
26	3.6
27	3.7
28	3.8
29	3.9
30	4
31	4.1
32	4.2
33	4.3
34	4.4
35	4.5
36	4.6
37	4.7
38	4.8
39	4.9
40	5

xp =

	0
25	57.98
26	60.285
27	62.59
28	64.895
29	67.2
30	69.507
31	71.814
32	74.123
33	76.434
34	78.747
35	81.062
36	83.38
37	85.701
38	88.025
39	90.352
40	92.683

yp =

	0
25	15.221
26	14.543
27	13.774
28	12.911
29	11.957
30	10.91
31	9.771
32	8.539
33	7.215
34	5.797
35	4.287
36	2.684
37	0.987
38	-0.802
39	-2.686
40	-4.662

βgr =

	0
25	-10.64
26	-12.816
27	-14.953
28	-17.044
29	-19.087
30	-21.075
31	-23.007
32	-24.88
33	-26.691
34	-28.441
35	-30.127
36	-31.751
37	-33.313
38	-34.813
39	-36.253
40	-37.633

El valor de xp correspondiente a yp=0 coincidirá con el alcance máximo del implemento. En este caso estará entre 85,70 y 88,02 m, lo cual se precisa en el gráfico de la trayectoria.

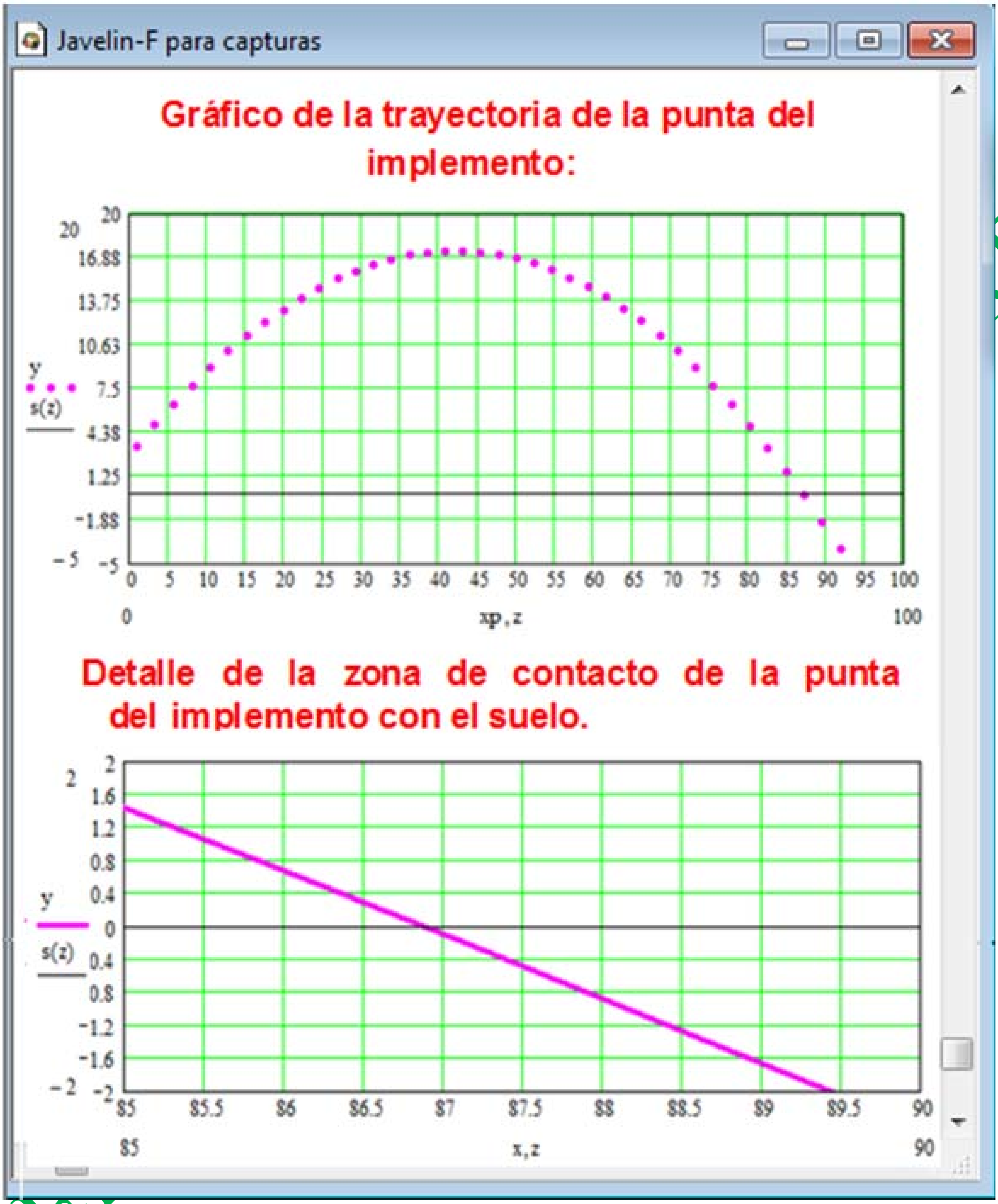
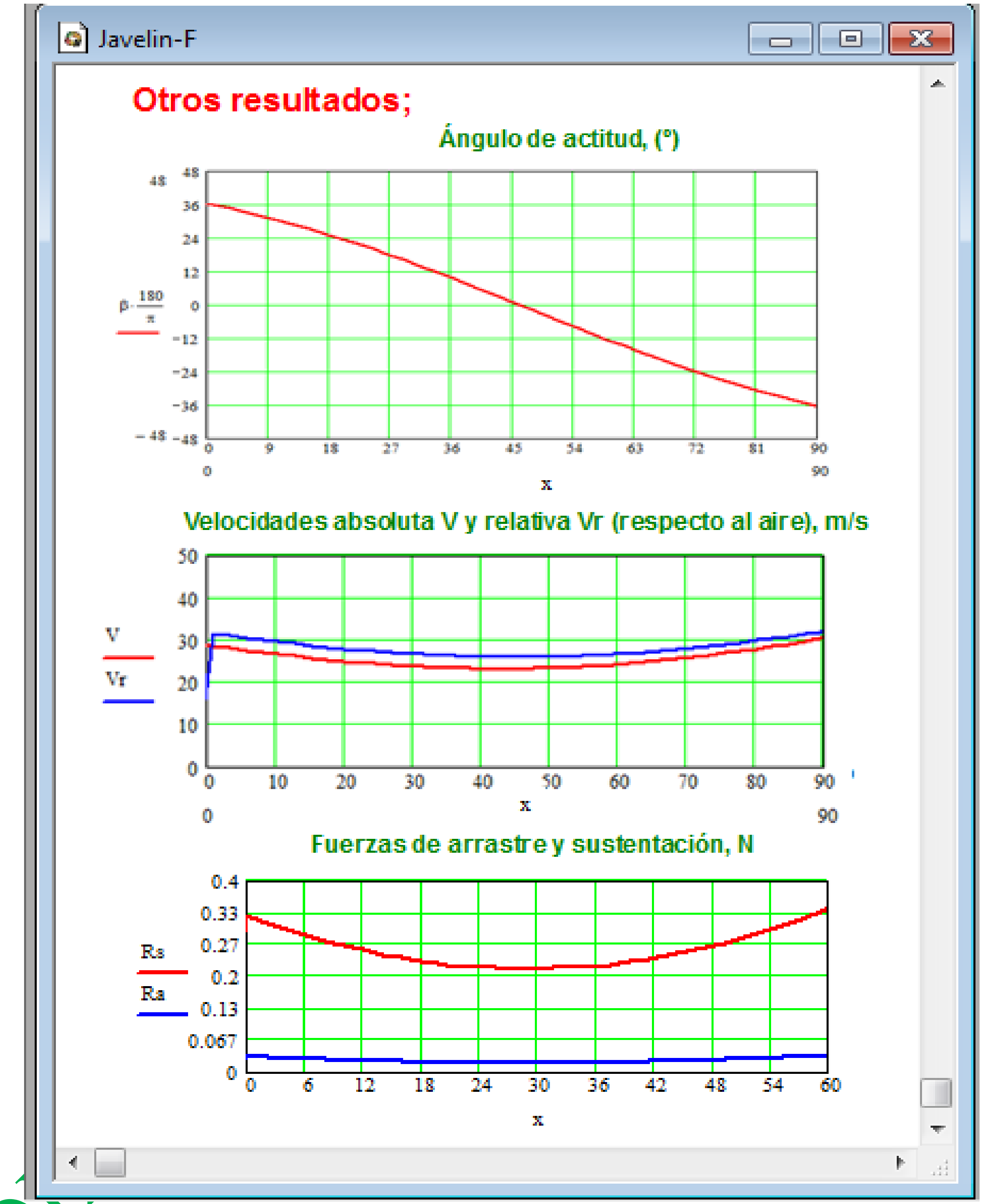


Gráfico de la trayectoria de la punta del implemento:

Detalle de la zona de contacto de la punta del implemento con el suelo.


Javelin-F
Otros resultados;
Ángulo de actitud, (°)
48
48
36
24
12
0
−12
−24
−36
−48
−48
$\beta \cdot \frac{180}{\pi}$
0
9
18
27
36
45
54
63
72
81
90
0
90
x
Velocidades absoluta V y relativa Vr (respecto al aire), m/s
50
40
30
20
10
0
V
Vr
0
10
20
30
40
50
60
70
80
90
x
0
90
Fuerzas de arrastre y sustentación, N
0.4
0.33
0.27
0.2
0.13
0.067
0
Rs
Ra
0
6
12
18
24
30
36
42
48
54
60
x